A stochastic scheduling algorithm for precedence constrained tasks on Grid

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\begin{abstract}
This paper addresses the problems in scheduling a precedence constrained tasks of parallel application with random tasks processing time and edges communication time on Grid computing systems so as to minimize the makespan in stochastic environment. This is a difficult problem and few efforts have been reported on its solution in the literature. The problem is first formulated in a form of stochastic scheduling model on Grid systems. Then, a stochastic heterogeneous earliest finish time (SHEFT) scheduling algorithm is developed that incorporates the expected value and variance of stochastic processing time into scheduling. Our rigorous performance evaluation study, based on randomly generated stochastic parallel application DAG graphs, shows that our proposed SHEFT scheduling algorithm performs much better than the existing scheduling algorithms in terms of makespan, speedup, and makespan standard deviation.

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\end{abstract}

1. Introduction

The popularity of the Internet and the availability of powerful computers and high-speed networks as low-cost commodity components make it possible to construct large-scale high-performance Grid computing systems. These technical opportunities enable the sharing, selection, and aggregation of geographically distributed heterogeneous resources for solving large-scale problems in science, engineering, and commerce [1,2]. To achieve the promising potentials of tremendous distributed resources, effective and efficient scheduling algorithms are fundamentally important. The scheduling problem deals with the coordination and allocation of resources as so to efficiently execute the users’ applications. Stochastic Grid parallel applications are submitted by users and generally independent of each other, which request systems’ services for their execution. The single parallel application is usually consisted of precedence constrained tasks that they generally require the use of different kinds of resources, e.g., computation, communication (network), storage resources, or specific instruments. A popular representation of a parallel application is the directed acyclic graph (DAG) in which the nodes represent application tasks and the directed arcs or edges represent inter-task dependencies, such as task’s precedence. In some cases, weights can be added to nodes and edges to express computational costs and communicating costs, respectively. Scheduling aims at meeting user demands (e.g., in terms of makespan, sum of weighted completion times, maximum lateness, and number of tardy jobs) and the objectives represented by the resource providers (e.g., in terms of profit and resource utilization efficiency), while maintaining a good overall performance (throughput) for the Grid computing systems [3]. It is widely known that the problem of finding the optimal schedule is NP-complete [4] in the general case. Therefore, heuristics can be used to obtain a sub-optimal scheduling rather than parsing all possible schedules. These methods may obtain suboptimal results, but they are much computational cheaper.

List scheduling is a very popular method for precedence constrained task scheduling based on the DAG model. The basic idea of list scheduling is to assign priorities to the tasks of the DAG and place the tasks in a list arranged in descending order of priorities. A task with a higher priority is scheduled before a task with lower priority, and ties are broken using some method. An important issue in DAG scheduling is how to rank (or weight) the tasks and edges (when communication delay is considered). The rank of a task is used as its priority in the scheduling. Once the tasks and edges are ranked, task-to-resource assignment can be found by considering the following two problems to minimize the makespan: how to parallelize those tasks having no precedence orders in the graph and how to make the time cost along with the critical path in the DAG as small as possible.

There are several effective Grid heuristics scheduling algorithms such as mapping heuristic (MH) [5], dynamic critical path(DCP) [6], levelized-min time (LMT) algorithm [7], dynamic-level scheduling (DLS) algorithms [8], critical-path-on-a-machine
COP) algorithms and heterogeneous earliest-finish-time (HEFT) algorithm [9–12]. The HEFT algorithm selects the task with the highest upper rank (an upward rank is defined as the maximum distance from the current node to the exiting node, including the computational cost and communication cost) at each step. The selected task is then assigned to the processor which minimizes its earliest finish time with an insertion-based approach which considers the possible insertion of a task in an earliest idle time slot between two already-scheduled tasks on the same resource. The time complexity of HEFT is $O(m \times v^2)$, where $m$ is the number of processors and $v$ is the number of tasks. HEFT algorithm significantly outperforms DLS, DCP, LMT, MH, and CPOP algorithms in terms of average makespan, speedup, etc., [9].

Most of the above researches assume that parameters such as task processing time and communication time between precedence constrained tasks are fixed and deterministic which are known advance. However, in real-world problems, it usually does not suffice to find good schedules for fixed deterministic processing time, since tasks usually contain conditional instructions and/or operations that could have different execution times for different inputs [13–16]. A natural step to tackle this problem is to consider stochastic scheduling, that is, to interpret processing time and communication time as random variables and to measure the performance of an algorithm by its expected objective value. In this paper, let $V(v_1, \ldots, v_m)$ be a set of tasks of a parallel application that have to be scheduled on heterogeneous Grid resources (or heterogeneous machines (or unrelated parallel machines) so as to minimize the schedule length (makespan). Any machine can process at most one job at a time, and every job has to be processed on one of the $m$ machines. In contrast to the deterministic version, the crucial assumption is that the task processing time $w(v_i)$ and communication time $w(v_{ij})$ are not known in advance. Instead, one assumes that the task processing time and communication time are random variables from which we are just given their distribution function. Throughout the paper, task durations are supposed to be stochastically independent.

Since there are a number of scheduling models with different conditions considered in stochastic scheduling [17–19,13,20,14,21–24], it is convenient to refer to them in the notation of Graham et al. [23] and Allahverdi et al. [24]. Each problem is denoted by the standard three-field notation $\alpha|\beta|\gamma$ with the following intended meaning.

- The field $\alpha$ specifies the machine environment. For instance, $\alpha = P$ denotes the model with identical parallel machines, $\alpha = Q$ denotes the problem where machines have different speeds $s_i$, and the processing time of task $v_i$ on machine $k$ is $w(v_i)/s_k$, and $\alpha = 1$ is used for problems with a single machine.
- The field $\beta$ contains the task characteristics. It can be empty, which implies the default of non-preemptive, precedence constrained tasks. Possible entries are, among many others, $\text{prec}$ for precedence constrained tasks, or $\text{pmtn}$ for preemptive tasks.
- The field $\gamma$ denotes the objective function. It is generally a function of the completion times of the tasks. For the total weighted completion time, we write $\gamma = \sum w_iC_i$. For the makespan $\gamma = C_{\text{max}}$.

As an example, $Q|v_i \sim \text{stoch}, \text{prec}|E[C_{\text{max}}]$ is the stochastic scheduling problem to minimize the makespan of precedence constrained tasks on Grid heterogeneous parallel machines. Precedence constraints between tasks play a particularly important role in most real-world parallel applications. Therefore, it would be both of theoretical and of practical interest to incorporate those constraints into stochastic scheduling. Considering stochastic tasks with precedence constraints, such as the stochastic DAG model, Skutella and Uetz obtained a performance guarantee of $(1 + \varepsilon)\left(1 + \frac{m - 1}{m} \max\{1, \frac{m - 1}{m}, \Delta\}\right)$ for $P|v_i \sim \text{stoch}, \text{prec}|E[w_iC_i]$ [here, $\varepsilon$ is an arbitrary constant] [21], where they assume that the squared coefficients of variation of all processing time $w(v)$ are bounded by some constant $\Delta$ that is expressed as $\operatorname{Var}(w(v))/E[w(v)]^2 \leq \Delta$, for all $v \in V$. For $\Delta \leq 1$, the performance guarantees can be simplified to $3 + 2\sqrt{2}/m$. In most case, the schedule length (makespan) is the key objective of stochastic scheduling. If all tasks are identically and exponentially distributed, Chandy and Reynolds show that a highest level first policy minimizes the expected makespan $E[C_{\text{max}}]$ for the special case of two machines and in-tree precedence constraints [22]. Here, the level of a task simply denotes the number of successors in the precedence graph. They also give counterexamples for the case of more than two machines. Bruno extends this result by proving that highest level first even minimizes the makespan stochastically [25]. Pinedo and Weiss extend Chandy and Reynolds result to the case where the exponential processing time distributions of tasks of different levels may be different [26]. For the case of more than two machines, Papadimitriou and Tsitsiklis prove asymptotic optimality of a highest level first policy [27]. Their assumptions are that all processing times are independent and identically distributed, and this distribution has finite moments of any order. This is particularly true for the exponential distribution. Although somewhat counter-intuitive, results for out-trees (out-forests) seem to be much more difficult to obtain: Coffman and Liu show that a preemptive highest level first policy is optimal to minimize the expected makespan for some rather restricted scenarios [28]. These results, however, are restricted to somewhat artificial precedence constrained tasks (such as in-tree precedence constraints) and execution on identical machines.

To the best of our knowledge, no results of scheduling algorithm were previously known for $Q|v_i \sim \text{stoch}, \text{prec}|E[C_{\text{max}}]$ on Grid with $m$ heterogeneous machines. In recognition of this, we drive a stochastic scheduling model for this problem. In order to effectively scheduling precedence constrained stochastic tasks, we propose a stochastic heterogeneous earliest finish time ( SHEFT) scheduling algorithm, which incorporate the stochastic attribute, such as expected value and variance, of task processing time and edge communication time into scheduling. At last, we establish a stochastic scheduling simulation experiment platform and compare SHEFT with two well-known scheduling algorithms HEFT and DCP.

The rest of the paper is organized as follows: In Section 2, we describe the definitions and background of stochastic scheduling problem on Grid computing systems. In Section 3, we propose a SHEFT scheduling algorithm. In Section 4, we verify the performance of the proposed algorithm by comparing the results obtained from performance evaluation and conclude the paper in Section 5.

2. Definitions and background

2.1. The Grid scheduling architecture

A Grid is a system of high diversity, which geographically distributed sites interconnect through WAN. We define a site as a location that contains many computing resources of different processing capabilities. Heterogeneity and dynamicity cause resources in grids to be distributed or in clusters rather than uniformly. We can also generalize a scheduling process in the Grid into two stages: resource discovering and filtering and resource selecting and scheduling according to certain objectives. As a study of scheduling algorithms is our primary concern here, we focus on the second step. Based on these observations, Fig. 1 depicts a model of Grid scheduling systems.

As Grid resources are under the control of local schedulers, which provide access point to the resources, the central Grid scheduler must be built on top of the existing local schedulers. Basically, a central Grid scheduler (GS) receives applications
from Grid users, selects feasible resources for these applications according to acquired information from the Grid Information Service module, and finally generates application-to-resource mappings, based on certain objective functions and predicted resource performance [29]. The central Grid scheduler is simpler to implement, easier to manage, and quicker to repair in case of a failure. Information about the status of available resources is very important for a Grid scheduler to make a proper schedule, especially when the heterogeneous and dynamic nature of the Grid is taken into account. The role of the Grid information service (GIS) is to provide such information to Grid schedulers. GIS is responsible for collecting and predicting the resource state information, such as CPU capacities, memory size, communication capacity, software availabilities, and load of a site in a particular period [29]. The launching and monitoring (LM) module implements a finally-determined schedule by submitting applications to selected resources, staging input data and executables if necessary, and monitoring the execution of the applications [29].

In our scheduling model, the Grid system is modeled as an undirected graph \( G = (V, E) \), where \( V \) is the set of \( v \) tasks that can be executed on any of the available machines; \( E \) is the set of directed arcs or edges between the tasks to represent the dependencies. For example, edge \( e_{ij} \in E \) represents the precedence constraint such that task \( v_i \) should complete its execution before task \( v_j \) starts its execution. A task may have one or more inputs. When all its inputs are available, the task is triggered to execute. After its execution, it generates its outputs. The weight \( w(v_i) \) assigned to task \( v_i \) represents its processing time and the weight \( w(e_{ij}) \) assigned to edge \( e_{ij} \) represents its communication time. In our stochastic setting, the processing time and communication time are random, assumed to be independent and exponentially distribution or normal distribution [20,14,26,27].

For each application submission, the user needs to estimate these probabilities or distributions, which can be obtained by building a historic table and using statistical profiling [14,16]. We think that statistical profiling can be used to determine probabilities in many applications in the Grid area.

The set \( \{v_j \in V : e_{ij} \in E\} \) of all direct predecessors of \( v_j \) is denoted by \( \text{pred}(v_j) \) and the set \( \{v_j \in V : e_{ij} \in E\} \) of all direct successors of \( v_j \) is denoted by \( \text{succ}(v_j) \). A node \( v \in V \) without predecessors, \( \text{pred}(v_i) = \phi \), is called an entry task and if it is without successors, \( \text{succ}(v_i) = \phi \), it is named exit task. Without losing of generality, we assumed that the DAG has exactly one entry task \( v_{\text{entry}} \) and one exit task \( v_{\text{exit}} \). If multiple exit tasks or entry tasks exist, they may be connected with zero time-weight edges to a single pseudo-exit task or a single entry task that has zero time-weight. Fig. 2 shows an example DAG with assigned task and edge which is exponential distribution.

### 2.2. Stochastic parallel application model

Task graphs are encountered in many models used in operations research and resource management, such as scheduling, computer, and network models. Graph models are often used also to study activities that contain some concurrency. For example, a DAG can describe parallel applications where nodes represent tasks and directed arcs represent synchronization constraints and communication among tasks. Generally, a stochastic parallel application is represented by a DAG \( G = (V, E) \), where \( V \) is the set of \( v \) tasks that can be executed on any of the available machines; \( E \) is the set of directed arcs or edges between the tasks to represent the dependencies. For example, edge \( e_{ij} \in E \) represents the precedence constraint such that task \( v_i \) should complete its execution before task \( v_j \) starts its execution. A task may have one or more inputs. When all its inputs are available, the task is triggered to execute. After its execution, it generates its outputs. The weight \( w(v_i) \) assigned to task \( v_i \) represents its processing time and the weight \( w(e_{ij}) \) assigned to edge \( e_{ij} \) represents its communication time. In our stochastic setting, the processing time and communication time are random, assumed to be independent and exponentially distribution or normal distribution [20,14,26,27].

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### 2.3. Scheduling attributes

The main difference between stochastic scheduling and deterministic scheduling is that processing time and communication time are deterministic or random. Thus, how to compute the probability distribution of tasks in the DAG graph is the key technique to stochastic scheduling. In this paper, the processing time and communication time are assumed to follow exponentially distributions, which is reasonable to most real-world problems [14,26,27].
In probability theory, the expected value of a random variable indicates its average or central value and reflects its most important attribute. However, the real value of random variable does not equal to its expected value in most case. The other important attribute of a random variable is variance, which is the expected value of the square of the deviation of that variable from its expected value. Thus, the variance is an important measure of the amount of variation within the values of that variable, taking account of all possible values and their probabilities. The variance is first examined by Möring and Schulz in stochastic scheduling problem and got a good performance approximation in paper [23]. At the same time, Skutella and Uetz pointed out that the real processing time of stochastic task is affected by the standard deviation of random variable. They prove that the system scheduling performance are bounded by some constant $\Delta$ that is expressed as $\text{Var}(X)/E[X]^2 \leq \Delta$ [26]. For $\Delta \leq 1$, the scheduling performance are mostly affected by the sum of random variable’s expected value and square of variance. For others, the scheduling performance increases as random variable’s variance decreases. Thus, we take the variance of random variable into account and the approximate weight of random variable $Aw(X)$ is defined as

$$
Aw(X) = \begin{cases}
E[X] + \sqrt{\text{Var}(X)} & \text{if } \text{Var}(X)/E[X]^2 \leq 1 \\
E[X] \left(1 + \frac{1}{\sqrt{\text{Var}(X)}}\right) & \text{otherwise}
\end{cases}
$$

(1)

As the processing time of tasks and communication time of edges in stochastic DAG are assumed to follow exponentially distributions, each random variable are satisfied with $\text{Var}(X)/E[X]^2 = 1$. Thus, the approximate weight of random variable with exponential distribution is $Aw(X) = E[X] + \sqrt{\text{Var}(X)}$. For example, the approximate weight of task $v_2$ in Fig. 1 is $Aw(v_2) = 0.67$.

Before presenting the objective function, it is necessary to define the approximate earliest execution start time $AS(v_i, p_k)$ and the approximate earliest execution finish time $AF(v_i, p_k)$ of task $v_i$ on machine $p_k$. For the entry task $v_{\text{entry}}$

$$
AS(v_{\text{entry}}, p_k) = 0.
$$

(2)

For the other tasks in the graph, the approximate earliest start time and earliest finish time are computed recursively, starting from the entry task, as shown in (3) and (4), respectively. In order to compute the approximate earliest start time for a task $v_i$, all immediate predecessor tasks of $v_i$ must have been scheduled:

$$
AS(v_i, p_k) = \max\{\text{Available}(p_k), \max_{v_j \in \text{pred}(v_i)}\{AF(v_j, p_k) + Aw(e_{i,j})\}\}
$$

(3)

$$
AF(v_i, p_k) = AS(v_i, p_k) + \frac{Aw(v_i)}{s_k}
$$

(4)

where $Aw(e_{i,j})$ is the approximate weight of edge $e_{i,j}$’s communication time which transferring data from task $v_j$ (scheduled on $p_j$) to task $v_i$ (scheduled on $p_k$) and is computed by Eq. (1). When both $v_j$ and $v_i$ are scheduled on the same machine, $Aw(e_{i,j})$ becomes zero, since we assume that the intramachine communication cost is negligible when it is compared with the intermachine communication cost. The $\text{pred}(v_i)$ is the set of immediate predecessor tasks to task $v_i$, and $\text{Available}(p_k)$ is the expected earliest time at which machine $p_k$ is ready for task execution. After all tasks in a graph are scheduled, the schedule length (i.e., overall completion time) will be the actual finish time of the exit task $v_{\text{exit}}$; thus, the schedule length (which is also called makespan) is defined as follows:

$$
\text{makespan} = \text{EFF}(v_{\text{exit}})
$$

(5)

where the $\text{EFF}(v_{\text{exit}})$ is the earliest actual execution finish time of task $v_{\text{exit}}$ and different from the approximate earliest finish time $AF(v_{\text{exit}})$. The objective function of the task-scheduling problem is to determine the assignment of tasks of a given application to machines such that its scheduling length is minimized. For ease of understanding, we summarize the notations and their meanings used throughout this paper in Table 1.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$</td>
<td>A set of $v$ stochastic tasks in the application</td>
</tr>
<tr>
<td>$v_i$</td>
<td>The $i$th stochastic task in the application</td>
</tr>
<tr>
<td>$w(v_i)$</td>
<td>The probability distribution of task $v_i$ processing time</td>
</tr>
<tr>
<td>$E$</td>
<td>A set of directed edges representing communication among tasks in $V$</td>
</tr>
<tr>
<td>$e_{i,j}$</td>
<td>The directed edge from $i$th task to $j$th task</td>
</tr>
<tr>
<td>$w(e_{i,j})$</td>
<td>The probability distribution of edge $e_{i,j}$ communication time</td>
</tr>
<tr>
<td>$p$</td>
<td>A set of $m$ machines</td>
</tr>
<tr>
<td>$p_k$</td>
<td>The $k$th machine in Grid</td>
</tr>
<tr>
<td>$s_k$</td>
<td>The $k$th machine computation capacity</td>
</tr>
<tr>
<td>$w(e_{i,j})$</td>
<td>The direct communication capacity between machines $p_k$ and $p_l$</td>
</tr>
<tr>
<td>$\text{succ}(v_i)$</td>
<td>A set of immediate successors of task $v_i$</td>
</tr>
<tr>
<td>$\text{pred}(v_i)$</td>
<td>A set of immediate predecessors of task $v_i$</td>
</tr>
<tr>
<td>$\text{AS}(v_i, p_k)$</td>
<td>The expected earliest execution start time of task $v_i$ on machine $p_k$</td>
</tr>
<tr>
<td>$\text{AF}(v_i, p_k)$</td>
<td>The expected earliest execution finish time of task $v_i$ on machine $p_k$</td>
</tr>
<tr>
<td>$\text{Aw}(X)$</td>
<td>The approximate weight of random variable $X$</td>
</tr>
</tbody>
</table>

Table 1 Definitions of notations.

3. The proposed stochastic list scheduling

This section presents an algorithm for list scheduling on Grid called SHEFT, which aims to achieve high performance and low complexity. The algorithm consists of two mechanisms, a list-mechanism, which is a modified version of the HEFT [9–11] heuristic scheduling algorithm and a machine assignment mechanism which each task (in order of its priority) is assigned to a machine that minimizes the makespan. The pseudo code of the SHEFT algorithm is shown in Algorithm 1. First, we outline the concept of stochastic task priority. Then, we propose and analyze the stochastic list scheduling algorithm.

Algorithm 1: The pseudo code for SHEFT algorithm.

1. Compute Srank value for all tasks use Eq. (6) by traversing application graph, starting from the exit task
2. Sort the tasks in a scheduling list by non-increasing order of Srank value
3. while there are unscheduled tasks in the list do
4. Remove the first task $v_i$ from the scheduling list
5. for each machine $p_n$ in the machine set ($p_n \in P$) do
6. Compute the approximate earliest start time use Eq. (3)
7. Compute the approximate earliest finish time by Eq. (4)
8. end
9. find the minimum approximate earliest finish time machine $p_n$
10. Assign task $v_i$ to machine $p_n$ with minimize $\text{AF}(v_i, p_n)$
11. end

3.1. Stochastic task priorities phase

Our stochastic list scheduling algorithm will use stochastic upward rank (Srank) attribution to compute tasks priorities. The Srank is explained in Definition 1.

Definition 1. Given a stochastic parallel application DAG with $v$ tasks and $e$ edges and Grid computing system with $m$
The maximum and minimum Height of the DAG vary with DAG size of task for HEFT and DCP algorithms. The comparison is intended to determine the deterministic one by giving the expected value as the weight of task for HEFT and DCP algorithms. The comparison is intended not only to present quantitative results, but also to qualitatively analyze the results and to suggest explanations, for a better insight in the overall scheduling problem.

To test the performance of these algorithms, we have built an extensive simulation environment of Grid systems with 16 heterogeneous machines that computation capacities varies from Pentium II to Pentium IV. In order to interconnect these machines, a network with switches is employed, where the network topology is randomly generated and each machine is randomly connected to a switch. The communication capacity of links are assumed to be uniformly distributed between 10 and 100 Mbits/s. Stochastic parallel application graphs are based on randomly generator by varying parameters such as DAG size and height of the DAG (h). The performance metrics chosen for the comparison are the schedule length (makespan Eq. (5)), the speedup, and the makespan standard deviation. The speedup is computed by dividing the sequential execution time (i.e., cumulative execution time) by the parallel execution time (i.e., the makespan of the output schedule) as shown in Eq. (8):

\[
\text{Speedup} = \frac{\sum_{v_i \in V} ET(v_i)}{\text{makespan}}
\]

where \( ET(v_i) \) is the actual execution time of task \( v_i \). The sequential execution time is computed by assigning all tasks to a single machine that minimizes the cumulative of the computation costs. If the sum of the computational costs is maximized, it results a higher speedup, but ends up with the same rank of the scheduling algorithms.

The other performance metric is makespan standard deviation. Intuitively, the standard deviation of the makespan distribution tells how narrow the makespan distribution is. The narrower the distribution, the smaller the standard deviation is. This metric is examined in this paper because when you are given two schedules the one for which the standard deviation is smaller is the one for which realizations are more likely to have a stable performance in the Grid system.

### 4.1. Randomly generated application graphs

In our study, we considered the randomly generated stochastic application graphs. A random graph generator was implemented to generates stochastic application DAGs with various characteristics that depend on several input parameters given below. Our simulation-based framework allows assigning sets of values to the parameters used by the random graph generator. This framework first executes the random graph generator program to construct the stochastic application DAGs, which is followed by the execution of the scheduling algorithms to generate output schedules, and, finally, it computes the performance metrics based on the schedules. For the generation of random graphs, which is a commonly used to compare scheduling algorithms [8,9], three fundamental characteristics of DAG are considered:

- **DAG size**: \( |v| \): The number of tasks in the application DAG.
- **Height of the DAG (h)**: The \( v \) tasks are randomly partitioned into \( h \) levels.
- **The maximum and minimum \( \lambda \) value** (\( \lambda_{\text{max}}, \lambda_{\text{min}} \)) of task processing time and communication time between tasks with exponentially distribution: The \( \lambda \) value is a uniform random variable on the interval (\( \lambda_{\text{max}}, \lambda_{\text{min}} \)).

In our simulation experiments, graphs are generated for all combinations of the above parameters with number of tasks ranged between 50 and 300. Every possible edge (DAGs are acyclic) is created with the same probability, which is calculated based on the average number of edges per task node. Every set of the above parameters are used to generate several random graphs in order to avoid scattering effects. The results presented below are the average of the results obtained for these graphs.

### 3.2. Machine selection phase

In machine selection phase, unscheduled task in the task sequence is selected and scheduled on a machine that can complete its execution with minimize approximate earliest finish time \( AF(v_i, p_s) \). To achieve this goal, SHEFT arranges task scheduling sequence by \( \text{Srank} \) that takes the attributes of random variable into account. For each stochastic task, SHEFT computes \( AF(v_i, p_s) \) by using Eqs. (3) and (4). These ideas are implemented from step 5 to 8. In step 9, SHEFT finds a machine with minimize approximate earliest finish time \( AF(v_i, p_s) \) and assigns task \( v_i \) to this machine \( p_s \) in step 10. It is to be noted that this machine may or may not be its best-suited machine due to the stochastic of task. This type of heuristic automatically takes into consideration the computing power of the candidate machine.

### 3.3. Algorithm complexity analysis

The time complexity of scheduling algorithms for parallel application DAG is usually expressed in terms of number of task \( v \), number of edges \( e \), and number of machines \( m \). The time-complexity of SHEFT is analyzed as follows. Computing the \( \text{Srank} \) of task can be done in time \( \max(O(v^2), O(vm)) \), and sorting the tasks can be done in time \( O(v \log v) \). The machine selection for all tasks can be done in time \( O(|v|v^2|m|) \), and \( |v| \) is the max degree of \( v_i \) in application DAG. In Grid systems, the number of machines may be larger than the number of application’s edges in most case. Thus, the complexity of the algorithm SHEFT is \( O(|v|v^2|m|) \).

### 4. Performance evaluation

In this section, we compare the performance of the SHEFT algorithm with two well-known scheduling algorithms in Grid systems: the HEFT [9] and DCP [6] algorithms. To make the comparison efficient, we turn the stochastic scheduling problem into deterministic one by giving the expected value as the weight of task for HEFT and DCP algorithms. The comparison is intended not only to present quantitative results, but also to qualitatively analyze the results and to suggest explanations, for a better insight in the overall scheduling problem.
4.2. Experimental results

The goal of the experiments is to compare the proposed SHEFT algorithm with HEFT and DCP. In order to effectively simulate the real-world stochastic environment, the actually execution time of task is evaluated by giving their random probability distribution. To make the comparison fair, each schedule of scheduling strategy has the same task random probability.

For the first set of simulation studies, we compare and analyse the sensitivity of machine number with 100, 200, and 300 tasks respectively. The results are shown in Figs. 3–5. For performance metrics of makespan and speedup, each data point is the average of the data obtained in 500 experiments, and for the makespan standard deviation metric, the data are from these 500 experiments. Fig. 3 shows the simulation results of the SHEFT, HEFT, and DCP algorithms on Grid system for stochastic parallel application DAG with 100 tasks. We observe from Fig. 3 that the SHEFT outperforms HEFT and DCP by (10.7%, 16.1%) in terms of the average makespan, (19.7%, 26.1%) in terms of the average speedup, and (21.3%, 9.97%) in terms of makespan standard deviation, respectively. This is mainly due to the fact that the SHEFT algorithm is a stochastic scheduling strategy and considers the stochastic attribute, such as expected value and variance, in scheduling algorithm, which produce a good schedule for stochastic scheduling problem $Q | v_i \sim \text{stoch}, \text{prec} | E[C_{\text{max}}]$. However, the HEFT and DCP algorithms are deterministic and...
ignore the available stochastic information about task processing time and communication time among tasks, which merely select a machine for a task with the expected value of variables. For example, the task processing time follows exponential distribution as $f(t) = 0.1e^{-0.1t}$, the expected value is 10, and the variance is 100. In this situation, the scheduling selections of HEFT and DCP are according to the value of 10, which is far from the actual execution time of task. However, the SHEFT schedules the task according to the value of 20, which is close to the actual execution time.

As the comparison between HEFT and DCP of simulation results, we find that HEFT is better than DCP in terms of makespan and speedup. This is mainly due to the fact that HEFT can more effectively compute the critical task than DCP in DAG. However, for makespan standard deviation, DCP is better than HEFT. This phenomena can attribute to the fact that the variance of DCP is better than HEFT. As the machine number increases, the makespan and makespan standard deviation of three algorithms decreases, and the speedup of them increases.

Fig. 4 shows the simulation results for stochastic parallel application DAG with 200 tasks. We observe from Fig. 4 that the SHEFT significantly outperforms HEFT by (30.7%, 39.05%, 9.8%) and DCP by (46.2%, 49.96%, 12.5%) in terms of the average makespan, speedup, and makespan standard deviation, respectively. However, the improvements of SHEFT over HEFT and DCP are almost at the same level except that the machine number is 14, which reflect the intricacy of stochastic scheduling problem. The improvements of SHEFT over HEFT and DCP could also be concluded from Fig. 5. Fig. 5 shows the simulation results for stochastic parallel application DAG with 300 tasks, SHEFT outperforms HEFT by (32%, 21.6%, 18.6%), and DCP by (34.3%, 29.1%, 9.7%) in terms of average makespan, speedup, and makespan standard deviation, respectively.

To examine the performance sensitivity of the three scheduling algorithms SHEFT, HEFT, and DCP to the stochastic parallel application DAG size, in this set of experiments, we vary DAG size from 50 to 300 with step 50. The results reported in Fig. 6 for 10 machines and Fig. 7 for 16 machines reveal that SHEFT outperform HEFT and DCP in terms of makespan, speedup, and makespan standard deviation. As the DAG size increases, the improvement becomes more significant. The above simulation results also show a fact that the deterministic scheduling algorithm (such as HEFT and DCP) is not suitable for stochastic scheduling problem $Q|v_i \sim \text{stoch}, \text{prec}|E[C_{\text{max}}]$.

5. Conclusions and future work

In this paper, we attempt to incorporate stochastic parallel application into task scheduling on Grid systems. We believe that it is mandatory to design and implement stochastic scheduling to meet the random of task processing time and edge communication time, which is a key attribute to the performance of stochastic scheduling problem $Q|v_i \sim \text{stoch}, \text{prec}|E[C_{\text{max}}]$. The problem is first formulated in a form of Grid system model and stochastic parallel application DAG. To solve the problem efficiently, a stochastic heterogeneous earliest finish time (SHEFT) scheduling algorithm is proposed that is a modified version of the deterministic scheduling algorithm (HEFT) and incorporate the stochastic attribute of task processing time and edge communication time into scheduling. The performance of stochastic heterogeneous earliest finish time scheduling algorithm was compared with deterministic HEFT and DCP. The comparisons were based on randomly generated application DAGs. The simulation experimental results clearly demonstrate that our proposed algorithm SHEFT outperforms the existing well-known scheduling algorithms HEFT and DCP in terms of minimizing the makespan, makespan standard deviation, and improving the speedup.

This work is one of the first attempts to investigate scheduling precedence constrained stochastic tasks problem into Grid system. Future studies in this domain are threefold: first, it will be interesting to extend our stochastic scheduling algorithm to consider more other constraints, such as deadline, release time etc.; second, it will be interesting to study the problem context under large-scale heterogeneous distributed computing systems with security and reliability requirements; third, how the average execution time and standard deviation of tasks are obtained is a challenging and interesting problem, we plan to study it in our future work.
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